Exploring Congruent Triangles

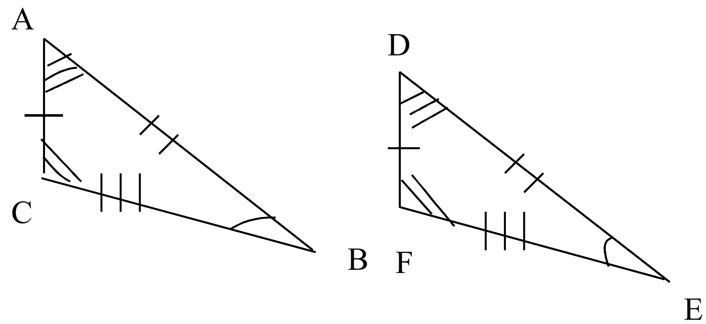
Triangles turn out to be very important shapes. The simplest possible polygons, triangles—particularly congruent triangles—are often the building blocks to proving facts about more complicated shapes.

The most important definition of this section is the following:

DEFINITION OF CONGRUENT TRIANGLES

Triangle ABC is congruent to triangle DEF if and only if

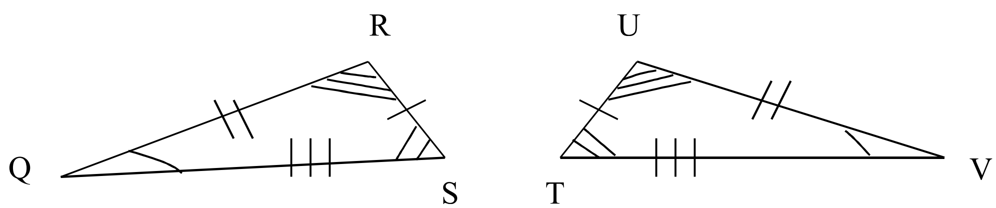
$\overline{AB} \cong \overline{DE}$	$\angle A \cong \angle D$
$\overline{BC} \cong \overline{EF}$	$\angle B \cong \angle E$
$\overline{AC} \cong \overline{DF}$	$\angle C \cong \angle F$



- Note that the order the triangles are named in (ABC and DEF) identifies exactly which angles are congruent and which sides are congruent.

- Note also the special notation on the diagram indicating which angles are congruent and which sides are congruent.

Practice: Name the congruence between the two triangles shown belo



Sometimes this definition is shortened to the following:

"Two triangles are congruent if and only if corresponding parts are congruent." This means *both* of the following statements are true:

1. If two triangles are congruent then their corresponding parts are congruent.
2. If two triangles have congruent corresponding parts then they are congruent triangles.

One of these statements will be used frequently in the proofs we do later in this section. Sometimes it is stated, "Corresponding parts of congruent triangles are congruent," or even shortened to "CPCTC." Identify which of the two statements above is equivalent to this phrase.

The other version of this definition is rarely used – we rarely attempt to prove two triangles are congruent by proving that all six pairs of congruent parts are congruent. The reason is that in many cases proving only three pairs of congruent parts is sufficient to guarantee that the triangles are congruent.

INVESTIGATING CONGRUENT TRIANGLES

In this activity we will investigate the implications of knowing various sets of three pairs of congruent parts. For example, we will consider what happens when two triangles are known to have congruent sides (the SSS condition). Are the triangles the exact same size and shape—in other words, are they congruent? Or is it possible to construct two triangles that are not congruent even though they have the same side lengths?

Exploration

ASA: Sketch a triangle with one angle of 30°, one angle of 70°, and an included side of 8 units.

SAS: Sketch a triangle with one side of 6 units, one of 8 units, and an included angle of 50°.

AAA: Sketch a triangle with angles of 40°, 30°, and 110°.

SSS: Sketch a triangle with sides of 6 units, 8 units, and 11 units.

SSA: Sketch a triangle with sides of 8 units, 10 units, and a *non*-included angle of 50°. (Hint—First sketch the 50°angle and measure off 10 inches on one of its rays. Now consider how to complete the triangle so it fits the given conditions.

Which combinations of congruent corresponding parts seem to *guarantee* congruent triangles?

Complete the following statements to summarize our findings:

Triangle Congruence Properties

1. ________: If the corresponding __________of two triangles are congruent then the triangles are congruent.
2. ________: If two pairs of corresponding __________ and the included __________ are congruent then the triangles are congruent.
3. ________: If two pairs of corresponding ___________ and the included __________ are congruent then the triangles are congruent.

TRIANGLE CONGRUENCE DIAGRAMS

Instructions: For each pair of triangles, if the two are congruent give an appropriate congruence statement and the applicable property (e.g., ΔABC ≅ ΔDEF by SAS). If it is not possible to prove congruence, so state. Note: Arrows indicate parallel segments.

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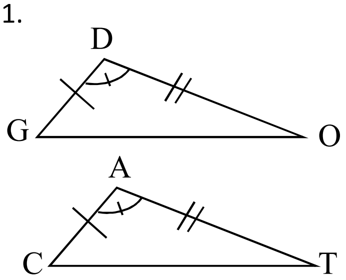

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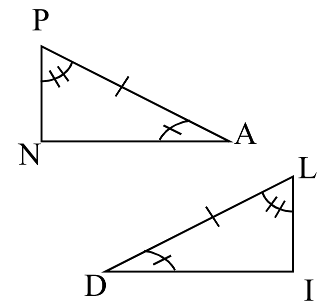

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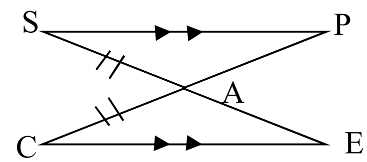

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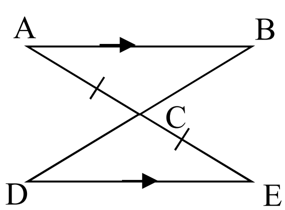

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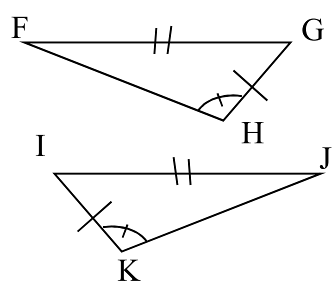

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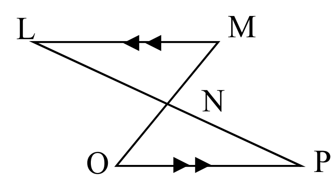

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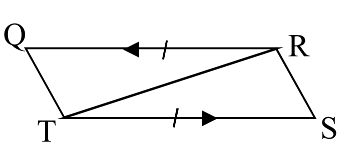

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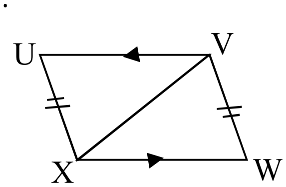

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